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ANALYTICAL ESTIMATES OF THE PP-ALGORITHM AT LOW NUMBER OF DOPPLER PERIODS PER PULSE LENGTH

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1. INTRODUCTION. When discussing the Doppler velocity estimators it is of significant interest to analyze their behavior at low number of Doppler periods $n_D=2V_r t_s/\lambda \approx 1$ within the resolution cell t_s (V_r is the radial velocity, λ-the wavelength). Obviously, at $n_D \lesssim 1$ the velocity error is essentially increased. The problem of low n_D arises in PBL, where higher resolutions are usually required but the signal-to-noise ratio (SNR) is relatively high. In this work analytical expression for the relative RMS error of the PP Doppler estimator at low number of periods for a narrowband Doppler signal and arbitrary model of the noise correlation function is obtained. The results are correct at relatively high SNR. The analysis is supported by computer simulations at various SNR's.

2. ANALYTICAL EXPRESSION FOR THE RELATIVE RMS ERROR. The sampled inphase and quadrature signals may be presented by [1]

$$I(i\Delta t)=A_i\cos\theta_i + b_i \;, Q(i\Delta t)= A_i\sin\theta_i + b_i'; \; i=1,n \qquad (1)$$

where $\Delta t=t_s/n$, A_i and $\theta_i=\omega . i\Delta t+\varphi_i$ are the amplitude and the phase of the signal, which is a stationary narrowband Gaussian process with variance σ^2, spectral width σ_s and mean Doppler frequency ω_o; the independent noise components b_i and b_i' have distribution $N(0,\sigma_b^2)$. If the condition $\sigma_s \ll \omega_o$ is fulfilled, we may assume that the amplitude and the phase of the signal are constant within the resolution cell and fluctuate from shot to shot. We also introduce Gaussian distribution $N(\omega_o,\sigma_\omega^2)$ for the averaged over the range gate frequency ω with $\sigma_\omega \ll \omega_o$. From $n_D \le 1$ it follows $\omega\Delta t \ll 1$. Then the relative estimate of the Doppler frequency fluctuations can be written in the form [1]:

$$\hat{\mu} = \hat{\omega}/\omega - 1 = \hat{x}/(1 + \hat{y}) \approx \hat{x}\,(1 - \hat{y} + \ldots), \qquad (2)$$

where $\hat{x}=\sum_i [A\alpha_i+\beta_i-\omega\Delta t(A\gamma_i+\delta_i)]/\omega\Delta t(n-1)A^2$, $\hat{y}=\sum_i (A\gamma_i+\delta_i)/(n-1)A^2$

$\alpha_i = b_{i+1}'\cos\theta_i + b_i\sin\theta_{i+1} - b_{i+1}\sin\theta_i - b_i'\cos\theta_{i+1}$; $\beta_i = b_i b_{i+1}' - b_{i+1} b_i'$

$\gamma_i = b_{i+1}\cos\theta_i + b_i\cos\theta_{i+1} + b'_{i+1}\sin\theta_i + b'_i\sin\theta_{i+1}$; $\delta_i = b_i b_{i+1} + b'_i b'_{i+1}$.
The term $\hat{y}$ in (2) can be neglected if SNR $\geq$ 5dB. To find the relative RMS error in (2), we average the noise, frequency and amplitude fluctuations, accepting the following approximations:

$$\omega = \omega_o + \delta\omega \quad , \quad 1/\omega \approx 1/\omega_o(1-\delta\omega/\omega_o)$$
$$\cos(s\omega\Delta t) \approx \cos(s\Omega) - s.\Delta t.\delta\omega.\sin(s\Omega)$$
$$\sin(s\omega\Delta t) \approx \sin(s\Omega) + s.\Delta t.\delta\omega.\cos(s\Omega) \ , \ \Omega = \omega_o\Delta t$$
$$A = A_o + \delta A \ ; \ A_o = \langle A\rangle \ ; \ 1/A \approx 1/A_o(1-\delta A/A_o + \delta A^2/A_o^2).$$

Finally, the general expression for the relative RMS error μ becomes:

$$\mu^2 = \frac{2.396\xi}{\Omega^2(n-2)^2}.\left\{M + \sum\left[P(s)\cos(s\Omega) + Q(s)\sin(s\Omega) + 4.168\xi R(s)\right]\right\} \qquad (3)$$

where $P(s) = 2bH(s) + k_1U(s) + k_2V(s)s$, $Q(s) = 4dH(s) + k_3U(s)s + k_4V(s)$,
$R(s) = aF(s) + \Omega^2 G(s)$, $U(s) = (n-2-s).[\rho(s+1)+\rho(s-1)]$,
$H(s) = (n-2-s).\rho(s)$, $V(s) = (n-2-s).[\rho(s+1)-\rho(s-1)]$,
$F(s) = (n-2-s).[\rho^2(s) - \rho(s+1).\rho(s-1)]$, $a = 1+\nu^2$, $b = 1+\nu^2+\Omega^2$,
$G(s) = (n-2-s).[\rho^2(s) + \rho(s+1).\rho(s-1)]$, $d = \nu^2\Omega$, $c = \Omega^2 - 1$,
$M = (n-2).\{a+b-\rho(1).\cos\Omega + \rho(1).(2\nu^2+1).(c.\cos\Omega - 2\Omega.\sin\Omega) +$
$+2.084\xi.[b.(1-\rho^2(1)) + 2\Omega^2\rho^2(1).(n-1)]\}$,
$k_1 = a.(b\cos\Omega - 2\Omega\sin\Omega) + d\Omega.\cos\Omega$, $k_2 = d.[b(\Omega\cos\Omega - 2\sin\Omega) - \Omega(a+1).\cos\Omega]$,
$k_3 = d.[b(2\cos\Omega + \Omega\sin\Omega) - \Omega(a+1).\sin\Omega]$, $k_4 = a.(2\Omega\cos\Omega + b\sin\Omega) + d\Omega.\sin\Omega$,
$\rho(s)$-normalized noise correlation function, $\xi = \sigma_b^2/\sigma^2$.
Using the above expression, we can analyze the behavior of the RMS error μ at different $n_D < 1\div2$ for an arbitrary noise model $\rho(s)$, varying the SNR and the number of samples $n = t_s/\Delta t$ within the interval t_s. The graphs of $\mu(n_D)$ at n=16,32 and 64 for the case of white noise and SNR=10 and 20 dB are given in Fig.1. As seen, the RMS error decreases at lower n. It is due to the increase of the phase step $\Omega = \omega t_s/n$ of the Doppler vector, while the white noise variance (or SNR) is not affected by n. There is a strong dependence of μ on SNR at lower n_D (Fig.2). At higher SNR $\approx$ 30 dB (as in the PBL), the relative error does not exceed 10% when $n_D \approx 0.5$ and 15% at $n_D \approx 0.25$. The graphs of μ on the radial velocity V_r for different resolutions ΔR=50, 100, 200 m (n=16) calculated by the above expressions are shown in Fig. 3. As seen, good resolutions may be achieved using PP-algorithm

when $n_D < 1$. At $V_r < 5$ m/s, the errors in the expressions (3) are essentially increased.

3. **COMPUTER SIMULATION.** At low SNR's and high n_D the neglection of the high-order terms in the estimate $\hat{\mu}$ decomposition would not be correct. In such cases, a computer simulation of the time series (1) has been performed with the number of pairs N=2000, 4000 and 6000 for a white noise at SNR ≥ 10, 0 and -10 dB respectively. Some of the results are shown in Figs. 4-6. In Fig. 4 the percentage of cases when PP technique leads to wrong velocity direction is plotted as a function of n_D for n=16. As seen, the probability of wrong velocity direction may be neglected for $n_D > 0.4$ at SNR ≥ 10 dB. The same may be concluded for SNR ≈ 5 dB and $n_D \geq 1$. In Fig. 5 the minimum values of n_D ensuring the relative RMS error of 10% (solid lines) and 20% (dashed lines) are plotted as a function of SNR. At 10 dB, the simulation gives 10%-RMS error at $n_D = 2$ for n=32 and $n_D = 1.5$ for n=16; μ=20% is provided at $n_D = 0.5 \div 0.7$. These values show that SNR=10 dB permits an accurate single-shot velocity measurement for $2V_r t_s/\lambda \approx 1$. At lower SNR's the $n_D > 2$ is required. The fraction of estimates with relative error below 10% vs the SNR at $n_D = 0.5$, 1 and 2 is presented in Fig. 6.

The comparison between the analytical and computer simulation results for $n_D \leq 1$ shows a good agreement at high SNR>10÷20 dB. It must be noted that the use of PP-algorithm at lower n_D is limited mainly by the relative error in velocity magnitude but not by the probability for wrong direction determination.

4. **CONCLUSIONS.** The results of this work show the relatively good performance of the PP-algorithm for Doppler frequency estimation at high SNR when better resolutions are required. The analytical expressions, obtained here may be used with different models of additive noises. The preliminary calculations show that at $n_D \leq 1$ the algorithm is strongly affected by the presence of correlated noises. Our further analysis of the expression (2) will be emphasized mainly on the effect of correlated noises, the presence of which is sometimes not easy to be controlled in the Doppler lidar channels.

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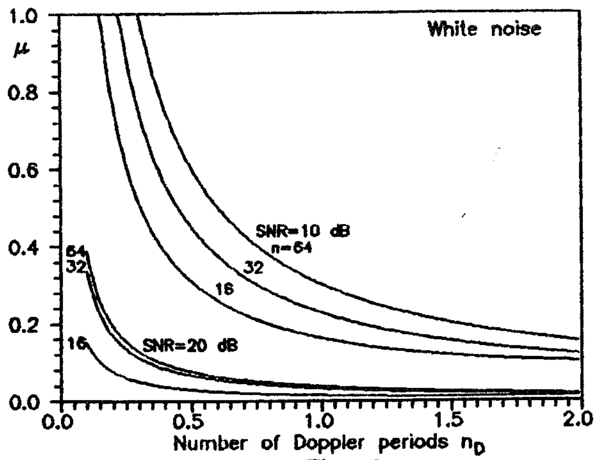

Fig. 1

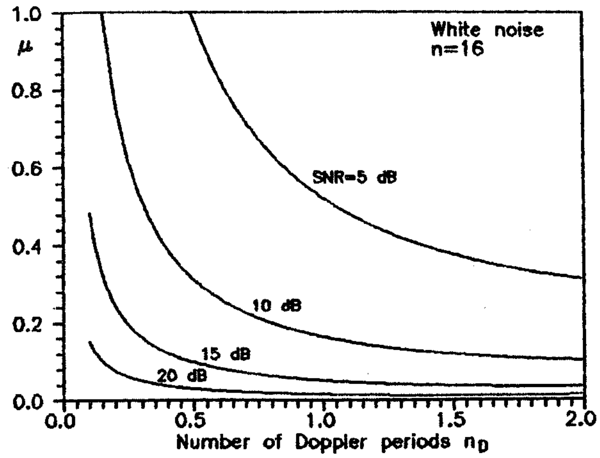

Fig. 2

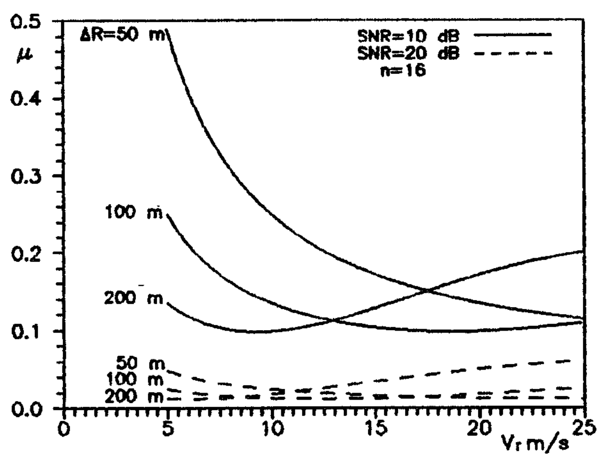

Fig. 3

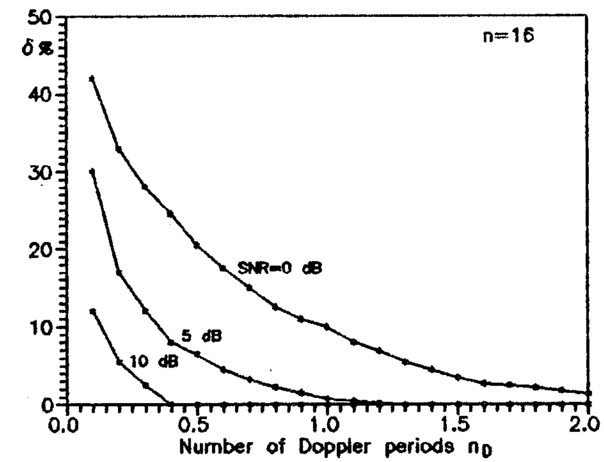

Fig. 4

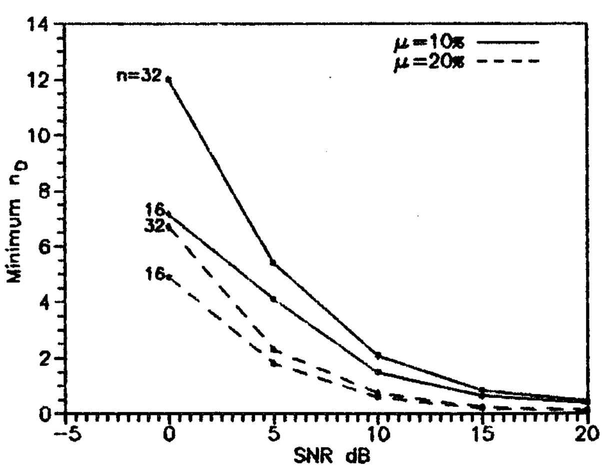

Fig. 5

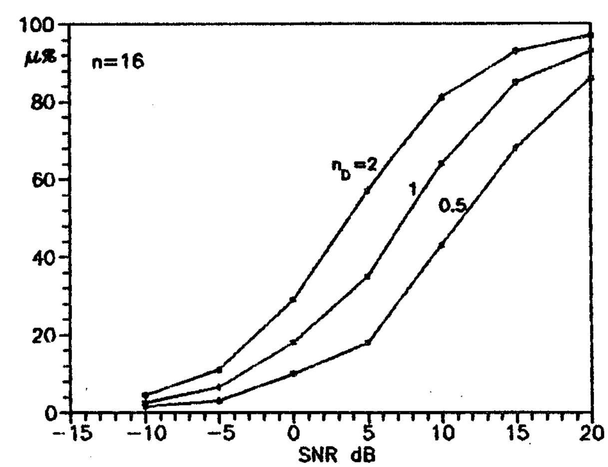

Fig. 6